THE EFFECT OF THE RELATIVE SPACING IN HEAT TRANSFER WHEN A BUNDLE OF TUBES IS IN THE TURBULENT FLOW OF A COOLANT ($Pr \geq 1$)

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We give the results of experimental investigations into heat transfer in a bundle of tubes with relative spacing $S/d = 1.1,\ 1.3,\ 1.4,\ 1.5$ in longitudinal water flow. A method of generalizing the experimental results and a computational expression for the heat transfer are proposed.

Surface elements with bundles of tubes in longitudinal flow are frequently encountered in the design of boilers and the heat transfer apparatus of atomic power plants. But at the present time there are no well-founded unified recommendations for calculating the heat transfer of such surfaces. The effect of the relative spacing of the tubes in the bundle on the heat transfer is still a matter for discussion [1]. To clarify this question the present paper describes a systematic investigation conducted on a single apparatus, of the heat transfer with water of four bundles differing in the relative spacings of tubes arranged in the form of an equilateral triangle ($S/d = 1.1,\ 1.3,\ 1.4$, and 1.5). The experiments were performed at atmospheric pressure with a circulation circuit made of steel Kh18N10T. Circulation was ensured by a TsND-6 centrifugal pump with a rating of 30 m^3/sec. The water flow rate was regulated by changing the rotational velocity of the motor shaft and was measured using a measuring nozzle which had previously been calibrated by the volume method. There were two variants of the working sections. In the first, bundles with relative spacing $S/d = 1.1,\ 1.3,\ 1.4$ [2] were prepared. The tubes of these bundles were heated by tubular heaters of Kh18N10T steel of length 800 mm, separated from the tubes of the bundle by an insulating layer ($\Delta \sim 1.5$ mm) of powdered boron nitride. Water current inlets of thick-walled copper tubes were welded to the heaters at both ends. The central calorimeter tube was made as follows: twelve grooves of dimensions 2.2×2.2 mm were milled on the surface of a copper tube of cross section 22×3 mm along the generator. Along the whole length of the tube copper capillaries of cross section 2×0.25 mm were fitted into the grooves. The capillary was soldered to the groove with brass solder, after which the tube surface was cleaned and polished. Into the open channels thus formed were introduced Chromel–Alumel thermocouples in capillaries of Kh18N10T steel of cross section 1×0.15 mm. The wall temperature of the calorimeter was measured along the 12 generators using these mobile thermocouples. The central tube was introduced into the bundle through a stuffing box. The heater is supplied with constant current from a low-voltage generator of type ANG-30. The bundle with relative spacing of $S/d = 1.5$ differed in

TABLE 1. Principal Geometric Parameters of the Bundles

Relative spacing of the bundle	Tube diameter d, mm	Number of tubes in bundle	Internal diameter of outer pipes, mm	Overall cross section F, mm^2	Heated length l, mm	Hydraulic diameter d_n, mm	Hydraulic diameter of cell d_∞, mm
1.1	22	7	76.8	1370	800	7.28	7.35
1.3	22	7	86.0	2845	800	14.85	18.9
1.4	22	7	107.0	5240	800	25.5	25.5
1.5	13	19	105.0	5365	1000	18.8	19.1

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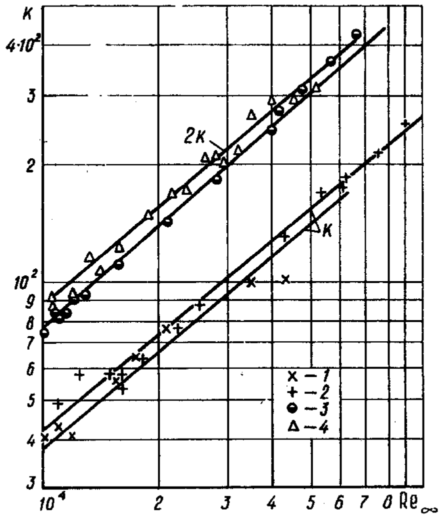

Fig. 1. Generalization of the experimental results of this paper on heat transfer in bundles in a longitudinal water flow in coordinates ($k = Nu_\infty/Pr^{0.4}(\mu_w/\mu_f)^{-0.11} = f(Re^\infty)$): 1) S/d = 1.1; 2) 1.3; 3) 1.4; 4) 1.5.

construction from those described above. All the heated tubes were isolated from the casing and alternating current from two ATMK autotransformers was supplied to them through an OSU-80. In this case the central calorimeter tube was of Kh18N10T steel. Chromel–Alumel thermocouples in quartz insulation were welded by electrical discharge to the inside wall of the calorimeter. After the thermocouples were welded, copper ends were soldered to the working part of the calorimeter. All the tubes of the bundle were heated in the course of the experiments. The principal geometrical parameters of the bundles are given in Table 1. The following quantities were measured in the experiments: mean mass water temperature at the inlet and exit of the working section, wall temperature of the calorimeter, electrical power supplied to the heaters of the bundle, and cooiant flow rate. The temperature of the sides of the tubes was not measured.

Heat balance was achieved for each of the experimental conditions. In most experiments the irreducibility in the heat balance did not exceed 5%. This irreducibility was slightly larger at small flow rates, but it did not exceed 10%. The experimental results were processed as follows.

1) From the water temperature at the inlet to the bundle and at the outlet of the central cells the change in the mean mass temperature of the liquid along the length of the working section was determined. The water temperature at the outlet of the central cells (t_0) was derived from the heat balance assuming that there was no mixing of the liquid between the cells.

2) The temperature head beyond the stabilization Δt_s was determined, the wall temperature of the calorimeter at each wall being taken as the mean in its neighborhood.

3) After making corrections taking account of the temperature difference between the outer surface of the calorimeter and the hot thermocouple junction, the stabilized value of the heat transfer coefficient α was calculated from the equation

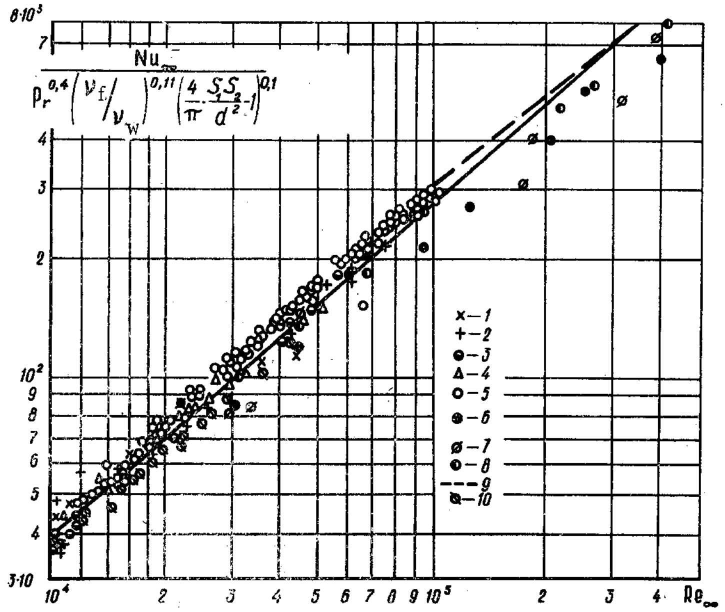

Fig. 2. Comparison of results on heat transfer in bundles in longitudinal water flow in coordinates ($Nu_\infty/Pr^{0.4}(\mu_f/\mu_w)^{0.11} \cdot [(4/\pi)(S_1S_2/d^2)-1]^{0.1} = f(Re_\infty)$): 1) S/d = 1.1; 2) 1.3; 3) 1.4; 4) 1.5; 5) 1.4 [3]; 6) 1.12 [6]; 7) 1.2 [6]; 8) 1.27 [6]; 9) 1.46 [5]; 10) 1.22 [4].

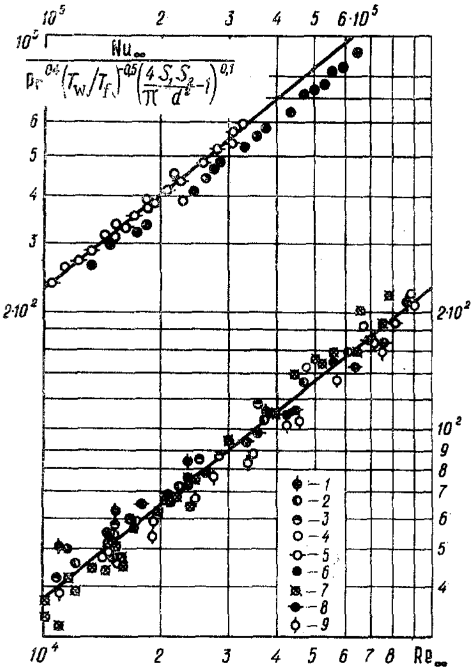

Fig. 3. Comparison of results on heat flow in bundles in longitudinal air flow in coordinates, $Nu_\infty/Pr^{0.4}(T_w/T_f)^{-0.5}[(4/\pi)(S_1S_2/d)-1]^{0.1} = f(Re_\infty)$: 1) S/d = 2 [8]; 2) $S_1/d = 2.36$, $S_2/d = 2.45$ [8]; 3) $S_1/d = 1.69$; $S_2/d = 1.64$ [8]; 4) S/d = 1.46 [10]; 5) S/d = 1.75 [10]; 6) S/d = 2.2 [10]; 7) S/d = 1.2 [11]; 8) S/d = 2.37 [9]; 9) S/d = 2.05 [9].

$$\alpha = \frac{q_c}{\Delta t_{stab}}. \quad (1)$$

The experimental data were presented in criterial form. Initially the definitive dimension was the hydraulic diameter of the bundle containing an infinite number of tubes (d_∞). In all bundles, apart from that which S/d = 1.3, $d_h \approx d_\infty$, and so the coolant flow rate could be determined as the mean flow (w = V/F). The physical properties of the water were referred to the mean liquid temperature

$$\bar{t}_f = \frac{t_{in} + t_{out.p}}{2}.$$

The effect of variation in the physical properties of the water across the cross section was estimated, by analogy with a circular tube, by introducing the correction $(\mu_w/\mu_f)^{-0.11}$. In most of the experiments this correction was scarcely different from unity.

Generalization of the experimental results in the form

$$\frac{Nu}{Pr^{0.4}\left(\frac{\mu_w}{\mu_f}\right)^{-0.11}} = f(Re) \quad (2)$$

showed that the experimental points lay on separate lines in accordance with the relative spacing (Fig. 1). Analysis showed that the experimental results could be generalized by introducing a new linear dimension* defined by the overall cross section of an individual cell S_{ce}:

$$d_r = \sqrt{\frac{4S_{ce}}{\pi}}. \quad (3)$$

For the bundles in triangular arrangements which we investigated

$$d_r \sim d_\infty \left[\frac{2\sqrt{3}}{\pi}\left(\frac{S}{d}\right)^2 - 1\right]^{-0.5}. \quad (4)$$

In the more general case of an arbitrary arrangement of the tubes, (4) becomes

$$d_r \sim d_\infty \left[\frac{4}{\pi}\cdot\frac{S_1S_2}{d^2} - 1\right]^{-0.5}, \quad (5)$$

where S_1/d and S_2/d are the longitudinal and transverse relative spacings in the plane of the cross section of the bundle. If we assume that $Nu \sim Re^{0.8}$, this replacement of the linear dimension is similar to the introduction of a correction in the relative spacing in the usual criterial equation of the form

$$\varphi = \left[\frac{4}{\pi}\cdot\frac{S_1S_2}{d^2} - 1\right]^{0.1}. \quad (6)$$

The introduction of the correction φ for $Re > 10^4$ "compresses" the experimental points practically onto a single straight line given by the equation

$$Nu = 0.015\,\varphi\, Re^{0.85} Pr^{0.4}\left(\frac{\mu_w}{\mu_f}\right)^{-0.11}. \quad (7)$$

It is interesting to observe that the heat transfer is a function of $Re^{0.85}$. To compare the results of the experiments described in this paper with those of earlier investigations [3-6, 16],† the latter were recalculated by a similar method. The recalculation proceeded as follows:

*This linear parameter is mainly used only for bundles which are not very closely packed, i.e., S/d ⩾ 1.1. It has been used for another case previously by Mikheev and Fedynskii.

†While the manuscript was in preparation, [16] appeared. Because the original data were not given, they could not be used in the comparison.

TABLE 2. The Dependence $Nu/Pr^{0.4} = (\xi/8)RePr^{0.6}/[4.5\sqrt{\xi}(Pr^{2/3} - 1) + 1.07]$

ξ	0,0316	0,0237	0,0182	0,0147	0,0124
			Re		
Pr	10^4	$3{,}2\cdot10^4$	10^5	$3\cdot10^5$	10^6
0,71	35,9	83,2	197	469	1271
1	36,2	86,5	210	506,6	1368
2	38,6	97,1	242	600	1610
3	38,0	95,8	244	616	1724
5	38,2	103,5	266	682	1840

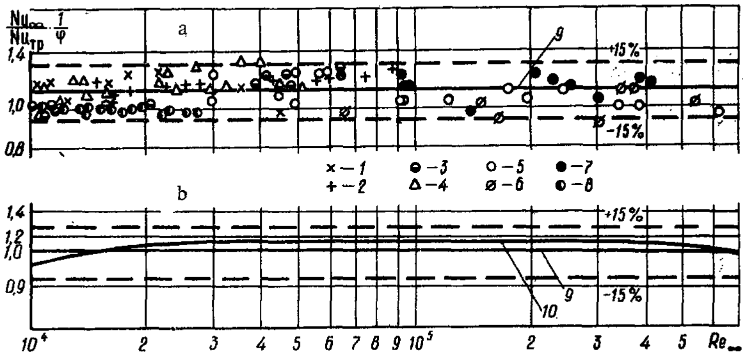

Fig. 4. Generalization of the results on heat transfer in tube bundles in water (a) and air (b) flow: 1) S/d = 1.1; 2) 1.3; 3) 1.4; 4) 1.5; 5) 1.12 [6]; 6) 1.2 [6]; 7) 1.27 [6]; 8) 1.22 [4]; 9) the line $(Nu_\infty/Nu_{tu})(1/\varphi) = 1.1$; 10) the line, $Nu_\infty = 0.02\,Re^{0.8}$.

1. The hydraulic diameters of the various types of cells were calculated from the cross section of the cell under consideration.

2. The equation $w_i/w_k \approx (d_{ri}/d_{rk})^{1/2}$, which gives, in the first approximation, the ratio of the velocities in cells in parallel, was used to determine the velocity of the central cells of the bundle, where the wall temperature of the calorimeter was measured.

3. In all possible cases (when there was sufficient data), the temperature head in the central cell of the bundle was refined in accordance with the calculated heating of the liquid.

4. The definitive dimension in the similarity criteria was taken as d_∞, the hydraulic diameter of a bundle with an infinite number of tubes. In Fig. 2 above data on heat transfer in water are compared with the results of the present investigation. It follows from the graph that our results agree well with those of [3]. The results of [6] lie somewhat below the line (7). The reason for the deviation of the points of [6] from the straight line is explained below. On the graph the results of [5] are shown as a mean (dashed) line.* The results obtained by various investigators for experiments with air [7-11] are constructed in Fig. 3 in the form of the equation

$$\frac{Nu}{Pr^{0.4}\varphi\left(\frac{T_w}{T_f}\right)^{-0.5}} = f(Re), \tag{8}$$

which, except corrections due to the variation in physical properties, are similar to those of [7]. The generalization includes the following domains of variation of the parameters:

$$10^4 < Re < 9\cdot10^5;\ 1.2 \leqslant S/d \leqslant 2.45.$$

*In the opinion of the authors, these results cannot be assumed to be sufficiently representative for the region of stabilized heat transfer [1].

The mean line is given by the equation

$$\mathrm{Nu} = 0.02\,\varphi\,\mathrm{Re}^{0.8}\left(\frac{T_w}{T_f}\right)^{-0.5}. \tag{9}$$

In comparing the results of analyzing the data for water and for air strict separation into lines was observed, which can be explained from an analysis of the relation between the heat transfer and the Prandtl number.

In analyzing the experimental results we used the relation $\mathrm{Nu} \sim \mathrm{Pr}^{0.4}$. However, if we refer to the results of a series of theoretical and experimental papers on heat transfer in turbulent flow in a circular tube [12, 15], they show that in the range of variation of Re and Pr such an approach is too coarse. Table 2 gives the results of calculations based on [13]. These calculations are confirmed by experimental data in [14, 15]. We adopt an analysis eliminating the effect of the Prandtl number:

$$\left(\frac{\mathrm{Nu}_\infty}{\mathrm{Nu}_{tu}}\right)_{\substack{\mathrm{Re=idem}\\ \mathrm{Pr=idem}}} = f\left(\frac{S_1}{d},\ \frac{S_2}{d}\right), \tag{10}$$

where we have taken the following expression [13] for Nu_{tu}:

$$\mathrm{Nu}_{tu} = \frac{\xi\,\mathrm{Re}\mathrm{Pr}}{4.5\sqrt{\xi}\,(\mathrm{Pr}^{2/3}-1)+1.07}, \tag{11}$$

and

$$f\left(\frac{S_1}{d},\ \frac{S_2}{d}\right) = k\varphi. \tag{12}$$

Here φ is obtained from (6), and k is the constant in (10). We find that

$$\left[\frac{\mathrm{Nu}_\infty}{\mathrm{Nu}_{tu}\varphi}\right]_{\substack{\mathrm{Re=idem}\\ \mathrm{Pr=idem}}} = k. \tag{13}$$

A similar analysis is given in Fig. 4 for air (b) and water (a). Experimental points from this paper and from [6] are given for water. It is clear that the results of [6] are markedly lower than the line 7 (Fig. 2), since in this paper the Prandtl numbers were much lower than in the experiments shown in Fig. 1, namely 1.18 and 1.75.* On the basis of Fig. 4 the constant in (13) is 1.1. Thus, the final equation for calculating the heat transfer with liquid ($\mathrm{Pr} \gg 1$) in turbulent flow in the intertube space of longitudinal bundles of relative spacing $1.1 \le S/d \le 2.45$ takes the form

$$\mathrm{Nu} = 1.1\varphi\,\mathrm{Nu}_{tu} \tag{14}$$

NOTATION

S	is the tube spacing in bundle;
d	is the tube diameter;
Δ	is the thickness of insulation layer;
F	is the overall section of bundle;
l	is the heated length of bundle;
d_h	is the hydraulic diameter;
d_∞	is the hydraulic cell diameter;
α	is the heat transfer coefficient;
q_c	is the specific heat flux through the surface of the central tube;
d_r	is the definitive dimension;
S_{ce}	is the overall cross section of an individual cell;
t_{in}	is the inlet liquid temperature;
t_f	is the mean mass temperature of liquid;
t_{out}	is the temperature at outlet from central cells;
Δt	is the liquid temperature heat at wall;
w	is the mean flow rate through bundle;
V	is the water bulk flow rate;
$\mu_{w,f}$	is the dynamic fluid viscosity referred to wall and fluid temperatures respectively;
φ	is the correction to relative spacing;
$T_{w,f}$	is the air temperature referred to wall and flow temperatures respectively;

*The graph does not show the results of [3] since they have the same Prandtl number as in our case (in order of magnitude) and the points coincide with those of this paper.

ξ is the hydraulic drag coefficient;
Pr is the Prandtl number;
Nu is the Nusselt number;
Re is the Reynolds number.

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